falls below the ideal result by a constant amount. Note that the correction term is smaller than that shown in the bracketed term of Eq. (5), however, because of the addition of Q_{13} to account for heat conducted into region 3.

An inspection of Eqs. (5) and (7) shows that film thickness effects are truly negligible only when $(t/\tau_g)^{1/2} \gg 1/Q_{12}$. In that instance alone Q_{23}, the ratio of thermal properties in the standard liquid and the backing material, follows directly from a ratio of the bridge output signals. If it is desirable or necessary to measure $(\rho c\lambda)^{1/2}$ of the substrate on a shorter time scale, finite film thickness must be accounted for, and this can be done to first order simply by comparing the slopes of straight lines fitted to bridge output records plotted vs $t^{1/2}$. The ratio of these slopes corresponds nearly to that of the theoretical first-order solutions and hence gives $1 + Q_{23}$ directly. [For analyses at time $(t/\tau_g)^{1/2} \lesssim 30$ the exact solution should be employed of course.] It is worth noting that, in view of the practical difficulty of obtaining a suitably accurate bridge balance prior to pulsing, it is good practice to use the slope technique even when film thickness effects are truly negligible.

The theory presented here was developed to complement a study of thermal accommodation processes utilizing the transient response of thin-film gages during a period of a few microseconds following exposure to a suddenly heated gas.[6] It was thus desirable to measure $(\rho c\lambda)^{1/2}$ of the film substrate over a comparable period accounting for the possible influence of film thickness effects. Accordingly, several pulse-heating experiments were conducted using five different chemically deposited and vacuum-evaporated platinum gages. The films, which varied in thickness from 400–2000 Å,† were all mounted on pyrex. The bridge circuit was carefully designed to minimize initial electrical transients during the pulsing process; a very sensitive bridge-balancing procedure was used. The bridge output signal for each experiment was monitored for either a 10 μsec or a 200 μsec period following pulse initiation. All the short- and long-duration records, when coupled with the present theoretical model, produced $(\rho c\lambda)^{1/2}$ values for pyrex of 3.60×10^{-2} cal-cm^{-2}-°K^{-1}-sec$^{-1/2}$ (at 25°C), with a maximum deviation of $\pm 0.13 \times 10^{-2}$. (The largest deviations occurred with the 10 μsec duration records.) Both silicone fluid (type MS 200/100 cs)‡ and glycerine were used as standard liquids, with the silicone fluid tests producing somewhat less scatter. Values for $(\rho c\lambda)^{1/2}$ of the liquids were computed from handbooks and manufacturer's literature. The value of $(\rho c\lambda)^{1/2}$ inferred for pyrex in this work is in excellent agreement with previous measurements[7] employing much longer time scales; so we may conclude that a) the value of $(\rho c\lambda)^{1/2}$ in pyrex is sensibly constant for the time scales of interest in shock-tube work, and b) there is no apparent variation of this value with the film materials or deposition processes employed here, even at very short times.

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Bending Stress in a Conical Shell Subjected to Thermal and Pressure Loadings

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Introduction

THIS Note employs the perturbation method of Ref. 1 to obtain the bending stress at a clamped support of a semi-infinite conical shell subjected to a spatially uniform heat addition that is constant through the thickness and has a step function in time. As in Ref. 2, the early time response is assumed to be adequately described by a model in which heat conduction is ignored. For a specific radius to thickness ratio, numerical results reveal that the maximum bending stress at the clamped support is significantly reduced by the first-order correction to the zero-order solution, which is that for a circular, cylindrical shell. Inclusion of the second-order correction, however, has a negligible effect and thus further correction terms are not computed.

In addition, the response of the conical shell to a spatially uniform pressure with a Heaviside temporal variation is obtained by integrating the solutions derived in Ref. 1. In contrast to the thermal loading, the bending stress at the clamped support for the pressure loading is increased when the first-order correction term is included. An additional point of interest is the similarity of the plots of the dimensionless bending stress at the support vs dimensionless time for the two loadings.

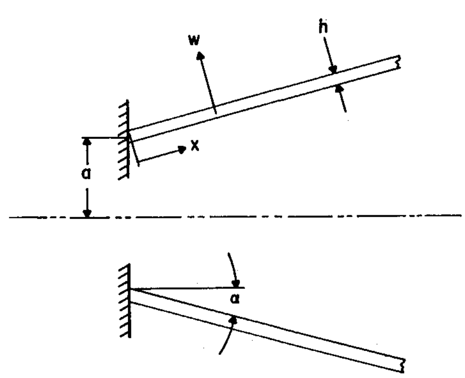

Fig. 1 Problem geometry.

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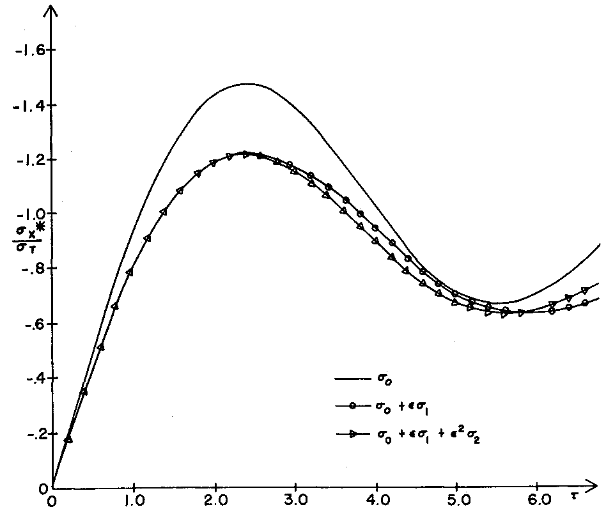

Fig. 2 Stress on outer shell surface due to heat addition for $\eta = 0$, $\alpha = 10°$, $a/h = 10$.

Formulation

For the present problem in which the stresses are thermally induced and the slant displacement terms in the equation of motion in the normal direction are assumed to be negligible, the expression governing the axisymmetric motion of a conical shell with slant coordinate x and half apical angle α (see Fig. 1) is readily obtained from Ref. 1. It is,

$$(1 + \eta \sin\alpha)^3 \frac{\partial^4 W}{\partial \eta^4} + 2 \sin\alpha[1 + \eta \sin\alpha]^2 \frac{\partial^3 W}{\partial \eta^3} - \sin^2\alpha[1 + \eta \sin\alpha] \frac{\partial^2 W}{\partial \eta^2} + \sin^3\alpha \frac{\partial W}{\partial \eta} + \frac{\cos^2\alpha}{b^2}(1 + \eta \sin\alpha)W + \frac{\cos\alpha(1 + \nu)[1 + \eta \sin\alpha]^2 \alpha^* T_0 H(\tau)}{b^2} = -\frac{(1 + \eta \sin\alpha)^3}{b^2} \frac{\partial^2 W}{\partial \tau^2} \quad \text{(1a)}$$

$$W = w/a, \quad \eta = x/a, \quad \tau = ct/a, \quad c^2 = E/\rho(1 - \nu^2), \quad b^2 = h^2/12a^2 \quad \text{(1b)}$$

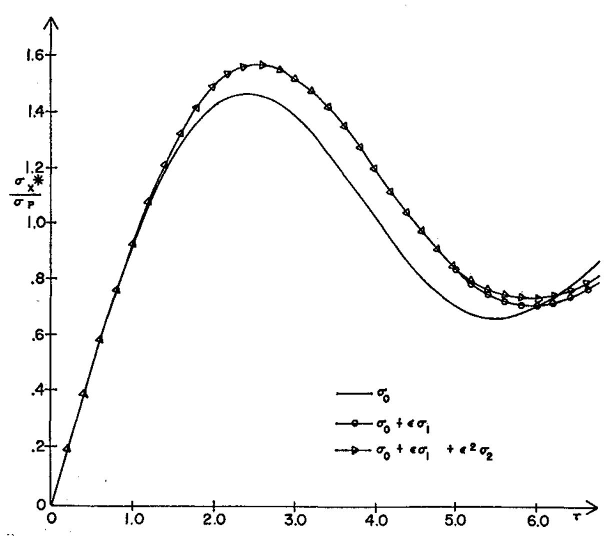

Fig. 3 Stress on outer shell surface due to pressure loading for $\eta = 0$, $\alpha = 10°$, $a/h = 10$.

where a and h are the shell radius at the clamped support and shell thickness; E, ν and ρ are Young's modulus, Poisson's ratio, and density, respectively; w is the normal displacement component, measured positive outward; t is time; α^* is the coefficient of thermal expansion, T_0 is the temperature rise, and $H(\tau)$ is the Heaviside step function.

From Ref. 1,

$$M_x = -Eh^3/12(1 - \nu^2)a\{(\partial^2 W/\partial \eta^2) + [\nu \sin\alpha/(1 + \eta \sin\alpha)] \partial W/\partial \eta\} \quad \text{(2a)}$$

$$\sigma_x^* = \pm 6M_x/h^2 \quad \text{(2b)}$$

Here M_x is the bending moment and σ_x^* the magnitude of the bending stress at the surfaces of the shell with the plus and minus signs referring to the radially outward and inward shell surfaces, respectively.

Defining

$$\epsilon = \sin\alpha \quad \text{(3)}$$

as the perturbation parameter and employing the perturbation procedure of Ref. 1 results in the following set of equations for a conical shell initially at rest in an undeformed configuration:

$$b^2(d^4\bar{W}_0/d\eta^4) + (1 + s^2)\bar{W}_0 = -(1 + \nu)\alpha^* T_0/s \quad \text{(4a)}$$

$$b^2 \frac{d^4\bar{W}_1}{d\eta^4} + (1 + s^2)\bar{W}_1 = -3\eta b^2 \frac{d^4\bar{W}_0}{d\eta^4} - 2b^2 \frac{d^3\bar{W}_0}{d\eta^3} - \eta(1 + 3s^2)\bar{W}_0 - 2\eta(1 + \nu) \frac{\alpha^* T_0}{s} \quad \text{(4b)}$$

$$b^2 \frac{d^4\bar{W}_2}{d\eta^4} + (1 + s^2)\bar{W}_2 = -3\eta^2 b^2 \frac{d^4\bar{W}_0}{d\eta^4} - 4\eta b^2 \frac{d^3\bar{W}_0}{d\eta^3} + b^2 \frac{d^2\bar{W}_0}{d\eta^2} + (1 - 3\eta^2 s^2)\bar{W}_0 - 3\eta b^2 \frac{d^4\bar{W}_1}{d\eta^4} - 2b^2 \frac{d^3\bar{W}_1}{d\eta^3} - \eta(1 + 3s^2)\bar{W}_1 - (1 + \nu) \frac{\alpha^* T_0}{s}\left(\eta^2 - \frac{1}{2}\right) \quad \text{(4c)}$$

where W is represented as

$$W(\eta,\tau) = W_0(\eta,\tau) + \epsilon W_1(\eta,\tau) + \epsilon^2 W_2(\eta,\tau) + \ldots \quad \text{(5)}$$

and $\bar{W}(\eta,s)$ is the transform of $W(\eta,\tau)$.

The transformed boundary conditions associated with the clamped support are

$$\bar{W} = d\bar{W}/d\eta = 0 \text{ at } \eta = 0 \quad \text{(6)}$$

The additional restriction on the solution is that $\bar{W}$ remain finite as $\eta \to \infty$. It should be noted that there is no restraint imposed on the slant displacement component at the support. For solutions in which the slant displacement component is zero, the boundary condition is that which is shown in Fig. 1. However, in both the present study and Ref. 1, valid solutions are also possible in which the slant displacement is nonzero and yet negligible compared to the normal displacement. In this case the clamped boundary is free to translate parallel to the cone axis.

The bending stress at the clamped support is readily obtained in a perturbation form from Eq. (2b) with the aid of Eqs. (2a), (5), and (6). Then solving Eqs. (4) and evaluating the second derivative of $\bar{W}_0$, $\bar{W}_1$ and $\bar{W}_2$ at $\eta = 0$ results in

$$\bar{\sigma}_x^*/\sigma_T = \pm(\bar{\sigma}_0 + \epsilon\bar{\sigma}_1 + \epsilon^2\bar{\sigma}_2 + \ldots), \text{ at } \eta = 0 \quad \text{(7a)}$$

$$\bar{\sigma}_0 = -1/s(1 + s^2)^{1/2} \quad \text{(7b)}$$

$$\bar{\sigma}_1 = -\frac{1}{2(3)^{1/4}}\left(\frac{h}{a}\right)^{1/2}\left[\frac{1}{s(1 + s^2)^{3/4}} - \left(\frac{15}{2}\right)\frac{1}{s(1 + s^2)^{7/4}} - \frac{10s}{(1 + s^2)^{7/4}}\right] \quad \text{(7c)}$$

$$\bar{\sigma}_2 = \frac{1}{8(3)^{1/2}}\left(\frac{h}{a}\right)\left(\frac{1}{s}\right)\left[\frac{-87}{(1+s^2)} - \frac{40}{(1+s^2)^2} + \frac{64}{(1+s^2)^3}\left(\frac{341}{256} + \frac{13s^2}{8}\right) + \frac{2b(-4+b^{-2})}{(1+s^2)^{3/2}}\ \frac{16b}{(1+s^2)^{5/2}}\right] \quad (7d)$$

where

$$\sigma_T = (3)^{1/2}E\alpha^* T_0/(1-\nu) \quad (7e)$$

Inverting Eqs. (7), the bending stress at the support is

$$\sigma_x^*/\sigma_T = \pm(\sigma_0 + \epsilon\sigma_1 + \epsilon^2\sigma_2 + \ldots), \text{ at } \eta = 0 \quad (8a)$$

$$\sigma_0 = -\int_0^\tau J_0(\lambda)d\lambda \quad (8b)$$

$$\sigma_1 = -\frac{(\pi)^{1/2}}{2(6)^{1/4}\,\Gamma(\frac{3}{4})}\left(\frac{h}{a}\right)^{1/2}\left[\int_0^\tau \lambda^{1/4}J_{1/4}(\lambda)d\lambda - 5\int_0^\tau \lambda^{5/4}J_{5/4}(\lambda)d\lambda - 10\int_0^\tau [\cos(\tau-\lambda)]\lambda^{1/4}J_{1/4}(\lambda)d\lambda\right] \quad (8c)$$

$$\sigma_2 = \frac{1}{256(3)^{1/2}}\left(\frac{h}{a}\right)\left[1336(\cos\tau - 1) - 649\tau\sin\tau - 75\tau^2\cos\tau + 64b(-4+b^{-2})\int_0^\tau \lambda J_1(\lambda)d\lambda - 174b\int_0^\tau \lambda^2(\lambda)d\lambda\right] \quad (8d)$$

where $\Gamma(\frac{3}{4})$ denotes the gamma function with argument $\frac{3}{4}$ and $J_n(\lambda)$ indicates a Bessel function of the first kind of order n. A tabulation of the integral appearing in Eq. (8b) is given in Ref. 3.

When the shell is subjected to a Heaviside pressure loading with amplitude P_o, the analogous stresses follow from an integration of the solutions for the impulsively loaded cone of Ref. 1. They are

$$\sigma_0 = \int_0^\tau J_0(\lambda)d\lambda \quad (9a)$$

$$\sigma_1 = \frac{(\pi)^{1/2}}{2(6)^{1/4}\,\Gamma(\frac{3}{4})}\left(\frac{h}{a}\right)^{1/2}\left[\int_0^\tau \lambda^{1/4}J_{1/4}(\lambda)d\lambda + (\tfrac{5}{3}) \times \int_0^\tau \lambda^{5/4}J_{5/4}(\lambda)d\lambda\right] \quad (9b)$$

$$\sigma_2 = \frac{1}{256(3)^{1/2}}\left(\frac{h}{a}\right)\left[(-280 + 75\tau^2)\cos\tau + 280 - 231\tau\sin\tau\right] + (\tfrac{1}{2})\int_0^\tau \lambda J_1(\lambda)d\lambda \quad (9c)$$

where the stress corresponding to σ_T is

$$\sigma_P = (3)^{1/2}(a/h)P_0 \quad (9d)$$

Results

Plots of the dimensionless bending stress on the outer shell surface at the clamped support vs dimensionless time with $\alpha = 10°$ and $a/h = 10$ are shown in Figs. 2 and 3 for the Heaviside heat addition and pressure loading, respectively. As indicated in Figs. 2 and 3, the first-order correction furnishes a significant contribution to the first peak value of stress. Further correction terms appear to be superfluous for the time interval of interest since the second-order correction is negligible during this time period. As in Ref. 1, the validity of the solutions for σ_1 and σ_2 decreases with increasing time due to the presence of secular terms in these equations.

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Use of Hot Wires in Low-Density Flows

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Nomenclature

A	= cross sectional area of hot wire
a	= defined Eq. (10)
C	= constant (function of $\bar{\alpha}$, d and l)
$F(S),f(S),g(S)$	= functions of molecular speed ratio S
I	= current
K_w	= thermal conductivity of wire at temperature T_w
k	= thermal conductivity of air
l	= length of wire
M	= Mach Number
Pr	= Prandtl Number
R	= Wire resistance
$\bar{R}$	= $(T_a - T_s)/(T_i - T_s)$
Re	= Reynolds Number
S	= speed ratio $(\gamma/2)^{1/2}M$
s	= defined Eq. (10)
T	= temperature
T_i	= temperature of wire at which there is no change in current in both the vacuum and flow
u	= velocity
α	= temperature-resistivity coefficient of wire, i.e. $\alpha = (R - R_r)/R_r(T - T_r)$
$\bar{\alpha}$	= accommodation coefficient
μ	= viscosity
ξ	= end loss parameter $(d/l)[(K_w/k_0)(1/Nu_0)]^{1/2}{}_{I\to 0}$
ν	= s/a
σ	= defined Eq. (3)
σ_1	= defined Eq. (4)
ρ	= density
ψ_N	= Nusselt Number correction factor

Subscripts

$(\ )_a$	= adiabatic
$(\ )_{corr}$	= corrected value
$(\ )_d$	= based on wire diameter
$(\ )_{fs}$	= freestream value
$(\ )_m$	= measured
$(\ )_0$	= evaluated at stagnation temperature
$(\ )_r$	= evaluated at reference temperature
$(\ )_s$	= hot wire support
$(\ )_{vac}$	= vacuum
$(\ )_w$	= evaluated at wire temperature
$(\ )_\infty$	= length-average of infinitely long wire

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